Magnetic field induced two-channel Kondo effect in multiple quantum dots

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We study the possibility to observe the two channel Kondo physics in multiple quantum dot heterostructures in the presence of magnetic field. We show that a fine tuning of the coupling parameters of the system and an external magnetic field may stabilize the two channel Kondo critical point. We make predictions for behavior of the scaling of the differential conductance in the vicinity of the quantum critical point, as a function of magnetic field, temperature and source-drain potential.

I. INTRODUCTION

Simple models for non-Fermi liquids, such as the multi-channel Kondo models, have attracted recently much theoretical interest. The predicted thermodynamic properties as well as transport properties are markedly different from the properties of a Fermi liquid. For example the conductance is predicted to approach its asymptotic zero temperature value as the square root of the temperature.

In this work we study a system built of a small quantum dot with even number of electrons coupled to a large dot and free leads. We explore the possibility to observe two channel Kondo physics in it by a fine tuning of the magnetic field and the couplings between the small quantum dot, the leads and the large dot.

In the first experimental realization of the *single channel Kondo* effect in planar quantum dot[1] transport was measured through a small dot containing odd number of electrons, which may be described by a local spin impurity model. At low temperature the spin is screened by the conduction electrons (of the single channel), a Kondo resonance is formed and the conductance through the dot approaches (in the symmetric dot case) to the universal value $G_0 = 2e^2/(2\pi\hbar)$ with a Fermi liquid T^2 law. Soon after the experimental realization of the single channel Kondo effect it was understood theoretically that in the presence of magnetic field[2] a *single channel <u>even</u> Kondo* resonance may be formed in a dot with an even number of electrons. For example, when there are two electrons in a two-level dot the relevant states are a singlet state $|00\rangle$ with total spin $S = 0$ and energy E_S; and a triplet of states, $|1S_z\rangle$ with $S = 1$ and $S_z = -1, 0, 1$ and energies $E_{T,S_z} = E_T + E_z S_z$, ($E_z = g\mu_B h$ is the Zeeman energy). If $E_S < E_T$ than the application of magnetic field reduces the energy of the triplet state with $S_z = 1$ until at $h_c \equiv (E_T - E_S)/g\mu_B$ the states $|1S_z = 1\rangle$, $|00\rangle$ are degenerate. The doublet of the degenerate state may be considered as isospin with which the conduction electrons form the Kondo state. Following these theoretical predictions the even Kondo effect was observed experimentally in vertical dots[3] and nanotubes[4].

It is important to appreciate that although the screening of the spin in both the single-channel Kondo effect and the single-channel-even Kondo effect is a non trivial many body phenomenon, eventually at low temperature, the spin is fully screened and both effects exhibit conventional Fermi liquid behavior. In contrast, when the dot has a doubly degenerate ground-state and there are two independent channels that equally screen the spin, an over-screening occurs and a non trivial many body two channel Kondo effect is formed. This many body state has both thermodynamic and transport properties that do not follow the predictions of the Fermi liquid theory.

The main difficulty in realizing a physical system that materializes the two channel Kondo model is in creating two separate channels that equally screen the spin. In conventional setups an electron from one channel that hops on the dot may hop to the other channel and causes mixing between the channels. This mixing lead eventually to two "eigen channels" with one channel coupled stronger than the other one. The channel with the stronger coupling fully screen the spin and the other channel is decoupled, and we thus have again the single channel Kondo case. It was suggested[5] to overcome this mixing problem by using a large quantum dot as an additional channel. Then, the free leads form one channel and the large dot forms the second channel (cf. Fig 1). The channels do not mix as transfer of electrons between them will charge the large dot. This suggestion for the realization of the *two channel Kondo effect* was realized experimentally[6].

In this work we combine the two channel Kondo effect with the even Kondo effect. We will show that it is possible to tune the magnetic field, the coupling strengthes of the small dot to the large dot and to the free leads in such a way that the two channel fixed point is stabilized.

The paper is arranged as follows: in section II we introduce a model that describes a small dot with even number of electrons connected to a large dot and free reservoirs. In subsections II A and II B we introduce a few simplifying assumptions concerning the symmetry of the dot – lead couplings, and define the appropriate vector oper-

ators **P** which describe the effective spin built from the singlet and one of the triplet states in the presence of accidental degeneracy induced by a magnetic field. Then in subsection II C we integrate out the high energy fields using a generalized Schrieffer – Wolff transformation until we obtain the effective low temperature Hamiltonian [Eq. (16)], which in general case demonstrates both channel and exchange anisotropy. As a special case we consider a situation when the small dot is split to two small dots. In section III we analyze the flow of the parameters using a Renormalization Group approach and discuss the conditions to observe the two channel Kondo physics. We find that in spite of the complicated form of Hamiltonian (16), the fixed point for two channel Kondo effect may be achieved by fine tuning the setup parameters. In section IV we discuss possible experimental realizations and observations of the predictions of the model. Some details of the calculations are relegated to appendices that appear after the summary.

II. MODEL

Generalizing the idea formulated in Ref. 5, we study a system consisting of a small dot d with two discrete energy levels labeled by index $m = 1, 2$ coupled with left L and right R leads and a large dot C, by means of tunneling amplitudes t_{mL}, t_{mR}, t_{mC}, respectively. (We have assumed here that the coupling constants do not depend on the state indices of the leads and the large dot.) The key feature of the model is that the dot C is sufficiently large so that the single particle mean level spacing δ in this dot is small enough to make its spectrum quasi continuous, $\delta \ll T_K$, whereas the Coulomb blockade energy E_C is large enough to fix the number of electrons $\mathcal{N}_C$, i.e. to prevent occupation of this dot by extra electron injected from the leads or from the small dot d.

We model the system (cf. Fig. 1) by the Hamiltonian $\mathcal{H} = \mathcal{H}_d + \mathcal{H}_C + \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{tun}}$, with

$$H_d = \sum_\Lambda E_\Lambda |\Lambda\rangle\langle\Lambda| \equiv \sum_\Lambda E_\Lambda X^{\Lambda\Lambda}$$

$$H_C = \sum_{k\sigma} \epsilon_{Ck\sigma} \hat{n}_{Ck\sigma} + E_C \mathcal{N}_C^2 \quad (\mathcal{N}_C = \sum_{k\sigma} \hat{n}_{Ck\sigma})$$

$$H_{\text{lead}} = \sum_{B=L,R} \sum_{k\sigma} \epsilon_k a_{Bk\sigma}^\dagger a_{Bk\sigma} \qquad (1)$$

$$H_{\text{tun}} = \sum_{k\sigma} \sum_{m=1,2} \sum_{A=B,C} \left(t_{mA} d_{m\sigma}^\dagger a_{Ak\sigma} + h.c. \right).$$

Here $a_{Bk\sigma}^\dagger, a_{Bk\sigma}$ are the creation and annihilation operators in the free left and right leads $B = L, R$, $a_{Ck\sigma}^\dagger, a_{Ck\sigma}$ are the creation and annihilation operators of the large dot C, $\hat{n}_{Ck\sigma} = a_{Ck\sigma}^\dagger a_{Ck\sigma}$, and the set of states $\{|\Lambda\rangle\}$ denote the exact many body eigenstates of the isolated small dot d. In our notation the eigen energies include not only the strong Coulomb blockade but also the Zeeman term, which do not influence the electrons
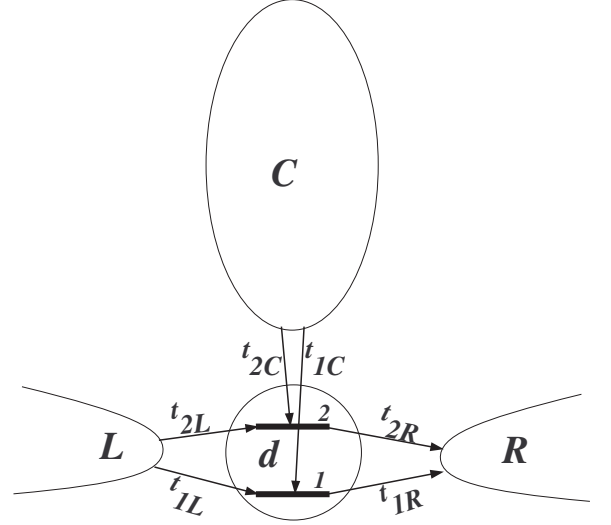

FIG. 1: A small dot d with even number of electrons coupled to a large dot C and two leads L and R. There are two levels in dot d. The charging energy U, in the small dot d is larger than the charging energy E_C in the large dot C.

in the free lead and the electrons in the large dot as by assumption they have a constant density of states. The operators $X^{\Lambda\Lambda} \equiv |\Lambda\rangle \langle\Lambda|$ are the Hubbard operators[7] (see Appendix A).

A. Symmetry properties of the tunneling matrix elements t_{mi}

When there is only one level in the dot, it is possible to perform a unitary transformation of the electron creation and annihilation operators in the left and right leads so that only one linear combination is coupled to the small dot, while the other one is decoupled[8]. In the generic case it is not possible to perform such rotation for the multilevel dot.[9] However, if the tunneling matrix elements do not depend on k and

$$\frac{t_{L1}}{t_{R1}} = \frac{t_{L2}}{t_{R2}} = \alpha \equiv e^{i\varphi} \tan\theta \qquad (2)$$

we can define:

$$a_{\Psi k\sigma} = e^{i\varphi} [\ \cos\theta a_{Rk\sigma} + \sin\theta a_{Lk\sigma}]$$
$$a_{\Phi k\sigma} = e^{i\varphi} [-\sin\theta a_{Rk\sigma} + \cos\theta a_{Lk\sigma}] \qquad (3)$$

which diagonalize the left-right sector of the Hamiltonian. To obtain H_{lead} in terms of $a_{\Psi k\sigma}$ and $a_{\Phi k\sigma}$ we simply have to substitute in Eq. (1) $B = \Phi, \Psi$, but the tunneling Hamiltonian is now:

$$H_{\text{tun}} = \sum_{k\sigma} \sum_{m=1,2} \sum_{A=\Psi,C} \left(t_{mA} d_{m\sigma}^\dagger a_{Ak\sigma} + h.c. \right). \qquad (4)$$

with

$$t_{1\Psi} = t_{1R}/\cos^2\theta, \quad t_{2\Psi} = t_{2R}/\cos^2\theta.$$

The Φ-channel is decoupled from the dot. Through out the paper we will assume that the symmetry relation (2) holds.

B. Projection to $|S\rangle$ $|T1\rangle$ subspace

The interesting physics will occur in a certain occupation of the small dot d. To understand what is the relevant subspace, we first assume that the strong Coulomb energy U fixes the number of electrons in the small dot to be even (say $\mathcal{N}_d = 2$). When the single particle level spacing in the small d-dot, denoted by δ, obey the inequality $t_{mi} \ll \delta \ll U$, we may further assume that only two single particle levels ϵ_m, $m = 1, 2$ in $\mathcal{H}_d$ are involved in the tunneling. So one can model the small dot d by a two-level dot with two electrons.

We denote the single particle levels of the dot by ϵ_1 and ϵ_2 and assume that $\epsilon_2 > \epsilon_1$. There are six possible states in this system, but we will neglect for simplicity the possibility of double occupation of level 2 and the singlet state of electrons occupying different levels. We thus remain with a subset of four lowest states: a spin singlet state, $|S\rangle$, with energy E_S when two electrons occupy level 1; and a spin triplet, $|T\rangle$, with energy E_T, when one electron occupy level 1 and the other level 2.

In terms of the creation operators with $d_{1\sigma}^\dagger$, $d_{2\sigma}^\dagger$ of electrons with spin σ at levels 1 and 2 respectively, the singlet state is given by

$$|S\rangle = d_{1\uparrow}^\dagger d_{1\downarrow}^\dagger |\text{vac}\rangle; \tag{5}$$

and the three triplet components by

$$
\begin{aligned}
|T_1\rangle &= d_{1\uparrow}^\dagger d_{2\uparrow}^\dagger |\text{vac}\rangle, \\
|T_0\rangle &= \frac{1}{\sqrt{2}} \left(d_{1\uparrow}^\dagger d_{2\downarrow}^\dagger + d_{1\downarrow}^\dagger d_{2\uparrow}^\dagger \right) |\text{vac}\rangle, \\
|T_{\bar{1}}\rangle &= d_{1\downarrow}^\dagger d_{2\downarrow}^\dagger |\text{vac}\rangle. \tag{6}
\end{aligned}
$$

The two-electron levels E_Λ are the eigenvalues of Hamiltonian H_d truncated in the way described above:

$$
\begin{aligned}
E_S &= 2\epsilon_1 + U \tag{7} \\
E_T &= \epsilon_1 + \epsilon_2 + U - J.
\end{aligned}
$$

We further assume that the exchange splitting J obeys the equation $\epsilon_2 - \epsilon_1 > J > 0$. In this case the singlet has a lower energy in the absence of magnetic field.

Next, in accordance with the standard approach[5,10], we neglect the finite level spacing δ_C in the large dot C, but suppose that the Coulomb blockade E_C fixes the number of electrons, $\mathcal{N}_C$, in the large dot. The interval δ_C denotes the lower limit of the energy of electron-hole excitation in dot C.

The Kondo tunneling regime is ineffective in our system unless $\delta = \epsilon_2 - \epsilon_1 < J$. This is a special situation, which may be achieved in some cases by tuning the gate voltage or playing with diamagnetic shift.[11,12] We

study here a general case of singlet ground state and appeal to the mechanism of accidental degeneracy induced by an in-plane external magnetic field h, which occurs when the Zeeman effect compensates the singlet–triplet splitting.[2,4,13] This mechanism is illustrated in Fig. 2. In our model the degeneracy between the states E_{T1} and E_S occurs when the condition

$$\delta - J = E_Z \tag{8}$$

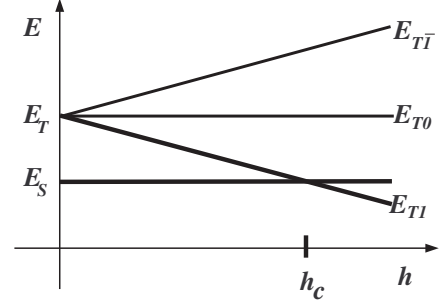is satisfied. Here $E_Z = g\mu_B h$ is the Zeeman splitting energy.

FIG. 2: Accidental degeneracy of a spin multiplet induced by Zeeman splitting.

To find out the possibility of a Kondo effect induced by an external magnetic field, one should derive the effective spin Hamiltonian containing relevant variables. Various procedures for the description of this phenomenon were discussed in Refs. 2,13. We will use the formalism of dynamical symmetry group, offered and elaborated in Refs. 14. According to this approach briefly described in Appendix A, the spin degrees of freedom in case of accidental degeneracy of the states $|S\rangle$ and $|T1\rangle$ are described by the vector $\mathbf{P}$ with components

$$
\begin{aligned}
P^+ &= |T1\rangle \langle S| &&\equiv f_\uparrow^\dagger f_\downarrow, \\
P^- &= |S\rangle \langle T1| &&\equiv f_\downarrow^\dagger f_\uparrow, \\
P^z &= \tfrac{1}{2} \left(|T1\rangle \langle T1| - |S\rangle \langle S| \right) &&\equiv \tfrac{1}{2} \left(f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow \right).
\end{aligned}
$$

The operator $\mathbf{P}$ obeys conventional commutation relations of momentum operator. It can be treated as an effective spin operator describing transversal and longitudinal spin excitations in a quantum dot occupied by two electrons under condition of accidental degeneracy illustrated by Fig. 2. We therefore may introduce an effective spin operator whose creation and annihilation operators with components $\sigma = \uparrow, \downarrow$ are denoted, respectively, by $f_\sigma^\dagger, f_\sigma$.

C. Integration of high energy states and the Schrieffer-Wolff transformation

To obtain the effective low energy Hamiltonian one has to integrate in the renormalization group sense the states

with high energy. There are few energy scales here: the charging energy of the small dot, U, the charging energy of the large dot E_C and the precise position of the many body levels in the dot. We will assume that this integration have renormalized the original parameters of Hamiltonian (1) and will concentrate on a situation when only the state $|00\rangle$ and $|11\rangle$ and single-electron states of the dot are relevant for virtual hopping between d and L, R and C. We will also assume that the scale of the energy considered is smaller than E_C so that transition between L and R to C (via d) are excluded.

After concentrating on the two states of the dot we are in a position to perform the Schrieffer-Wolff[15] transformation. The Schrieffer-Wolff transformation leads to terms of the form:

$$\sum_{\sigma,\sigma'=\uparrow,\downarrow \ A,A'=\Psi,C;} J_{AA'}^{ss'} c_{A\sigma}^\dagger c_{A'\sigma'} f_{\sigma'}^\dagger f_\sigma. \tag{9}$$

In the general case $J_{AA'}^{\sigma\sigma'}$ is a tensor of rank 6. However, due to the symmetry assumption (2) it reduces to a rank 4 tensor. For example the term $J_{CC}^{\uparrow\uparrow}$ describes a virtual transition from the triplet state to the large dot C and back. This transition occurs when an electron hops from the large dot C on the small dot d and back, or when one of the electrons that build the triplet state of dot d hops on dot C and back. According to our previous assumptions the former process is negligible and there are two contributions to the latter process. In the first an electron from level 1 hop on dot C and back and in the second electron from level 2 hop on dot C and back. Similar processes should be taken into account for $J_{\Psi\Psi}^{\sigma,\sigma'}$.

We have therefore:

$$J_{AA}^{\uparrow\uparrow} = \sum_{m=1,2} \frac{|t_{mA}|^2}{\epsilon_m + U - J - E_Z/2}. \tag{10}$$

Here the exchange integral is estimated for the intermediate state with the electron escaped from the dot d to the Fermi level which is supposed to be the same for the leads and the dot C, $\epsilon_{F\uparrow} = \epsilon_{F\downarrow} = 0$. The SW Hamiltonian is derived under the condition:

$$\epsilon_2 - J - E_Z = \epsilon_1. \tag{11}$$

Using this degeneracy condition, we derive

$$J_{AA}^{\downarrow\downarrow} = \frac{|t_{1A}|^2}{\epsilon_{1A} + U + E_Z/2} \tag{12}$$

$$J_{AA}^{\uparrow\downarrow} = \left(J_{AA}^{\downarrow\uparrow}\right)^* = \frac{t_{1A}t_{2A}^*}{\epsilon_1 + U + E_Z/2}.$$

Notice that due to the presence of magnetic field the $SU(2)$ symmetry is broken and $J_{AA}^{\sigma\sigma'}$ depend on σ and σ'. In the spirit of usual SW approximation, one may neglect the small differences in the denominators of above

exchange integrals and estimate them as

$$J_{AA}^{\uparrow\uparrow} = \frac{|t_{1A}|^2 + |t_{2A}|^2}{\epsilon_1 + U} \tag{13}$$

$$J_{AA}^{\downarrow\downarrow} = \frac{|t_{1A}|^2}{\epsilon_1 + U}, \quad J_{AA}^{\uparrow\downarrow} = \frac{t_{1A}t_{2A}^*}{\epsilon_1 + U}.$$

Due to the charging energy E_C, transitions between the Ψ, and C are excluded so that $J_{\Psi C}^{\sigma\sigma'} = 0$, in addition $J_{\Phi A} = J_{A\Phi} = 0$, $A = \Phi, \Psi, C$ since it was decoupled from the dot due to the symmetry assumption (2).

Rearranging the terms in (9), we may write the Hamiltonain in terms of the components of the vector $\mathbf{P}$ and the conduction electron spin $\sigma(\mathbf{x} = \mathbf{0})$ (In the general case H_{SW} has a more complicated form)

$$H_{SW} = \sum_{A=\Psi,C} \frac{J_{\perp A}}{2}\left(P^+\sigma_A^- + P^-\sigma_A^+\right) \tag{14}$$

$$+ \sum_A J_{zA}P^z\sigma_A^z + \sum_A J_{0A}P^z\sigma_A^0 + H_{\text{potential}}$$

where $H_{\text{potential}}$, describes potential scattering of conduction electrons without spin flips, $\sigma_{A\alpha\beta}^0 \equiv n_A/2 = \sum_{kk'} c_{Ak\alpha}^\dagger c_{Ak\beta}\delta_{\alpha\beta}/2$ and

$$J_{\perp A} = 2J_{AA}^{\uparrow\downarrow}$$
$$J_{zA} = J_{AA}^{\uparrow\uparrow} + J_{AA}^{\downarrow\downarrow}$$
$$J_{0A} = J_{AA}^{\uparrow\uparrow} - J_{AA}^{\downarrow\downarrow} \tag{15}$$

The total effective Hamiltonian H_{eff} contains an additional term $\propto P_z n_A$, which does not exist in the standard two channel Kondo Hamiltonian.[16] In the presence of magnetic field we may write it as:

$$H_{\text{eff}} = \sum_A \frac{J_{\perp A}}{2}(P^+\sigma_A^- + P^+\sigma_A^-) + J_{zA}P^z\sigma_A^z$$

$$+ \sum_A J_{0A}P^z(n_A - \bar{n}_A) + B_Z P^z + H_{\text{lead}}. \tag{16}$$

Here the first two terms describe an effective *multichannel* Kondo Hamiltonian, which is anisotropic both in spin and channel variables and includes effective magnetic field B_Z. This "field" is small in comparison with the Zeeman field E_Z (see Fig. 2),

$$B_Z = \frac{E_{T1} - E_S}{2} + \sum_A \frac{J_{0A}}{2}\bar{n}_A. \tag{17}$$

Here $\bar{n}_A$ is the average electron density in channel A. It is seen from (13) that an exchange anisotropy arises from the fact that there are two tunnel matrix elements t_{1A}, t_{2A} in H_{tun} [see Eq. (4)], which enter the longitudinal and transversal exchange constants in different combinations.

The exchange anisotropy may be eliminated from H_{SW} if the experimental setup is modified in accordance with Fig. 3. In this modification the small dot is split into
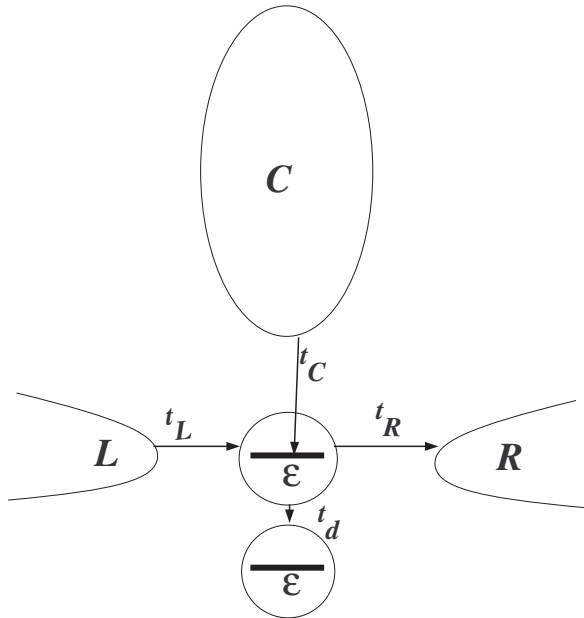
FIG. 3: Experimental setup with a small dot split into two equivalent wells.

two valleys coupled by tunneling element t_d. Each of these valleys is assumed to be occupied by odd number of electrons. If the highest unoccupied levels are equivalent, then the levels $\epsilon_{1,2}$, which enter the Hamiltonian H_d are

$$\epsilon_{1,2} = \epsilon \mp t_d; \qquad (18)$$

the singlet-triplet splitting in Eqs. (7) is determined by the indirect exchange $J' = 2t_d^2/U$ instead of direct exchange J; and the SW procedure described above gives an effective Hamiltonian in which only one tunnel matrix element, t_C, enters all parameters $J_{CC}^{\sigma\sigma'}$ (13). In contrust to the model of Fig. 1, with the split dot we obtain the standard two-channel Hamiltonian with isotropic exchange coupling (and no J_{0A}) in an effective magnetic field,

$$H_{\text{eff}} = \sum_A J'_A \mathbf{P} \cdot \boldsymbol{\sigma}_A + B'_Z P^z \qquad (19)$$

where B'_Z is the effective Zeeman field given by the first term in Eq. (17).

III. RENORMALIZATION GROUP EQUATIONS FOR EFFECTIVE HAMILTONIAN

To derive the flow equations for exchange vertices in a two-channel Hamiltonian, the conventional scaling procedure is used[17,18] and 3-rd order diagrams that contain loops are included.[16] We start with the setup of Fig. 3 which is described by the Hamiltonian (19) with isotropic exchange. In this case the system of flow equations has

the form

$$\frac{dj_a}{d\eta} = -j_a^2 + j_a(j_a^2 + j_b^2) \qquad (20)$$

where $b = \Psi$ if $a = C$ and vice versa, $j_a = \rho_a J_a$, ρ_a is the corresponding density of states on the Fermi level and $\eta = \ln D$ is the current scaling variable with initial value $D = D_0$. In accordance with the Nozieres-Blandin scenario, the flow diagram (j_Ψ, j_C) for this system has a weak coupling fixed point, which is unstable against channel anisotropy.

The system (20) is written for the case of $B_Z = 0$. It is known that the external magnetic field is a relevant parameter for the conventional two-channel Kondo effect.[19] In our case the two-channel regime arises only at certain magnetic field $h = h_c$ (Fig. 2) and, similar to the effect of an external magnetic field in conventional two channel Kondo system, the deviation $h - h_c$ from this critical value is detrimental for the two-channel physics.

The system of flow equations for the anisotropic Hamiltonian (16) with dimensionless coupling constants j_a^ι is more complicated. In this case the third-order corrections in the r.h.s. of scaling equations contains the sums $j_a^\iota \sum_b \sum_{\kappa \neq \iota} (j_b^\kappa)^2$, where a, b are the channel indices and the indices ι, κ mark the Cartesian components x, y, z in spin space. Besides, additional scaling equations should be written for the integrals j_{0a}.

It is easily seen that the vertices $j_a^0 = \rho_a J_{a0}$ remain unrenormalized in 2-nd order, and the third-order corrections enter with plus sign in the r.h.s. of corresponding scaling equations. Thus, the parameters j_a^0 scale into zero. As to the anisotropic third order terms in scaling equations for j_a^i, it is known from predictions of conformal field theory[20], that the exchange anisotropy is irrelevant for two-channel regime. So we conclude that the flow diagram for the two-channel Hamiltonian (16) with anisotropic exchange coupling retains the finite isotropic fixed point and only the channel anisotropy is relevant.

IV. EXPERIMENTAL REALIZATION AND CONSEQUENCES

A. Setup

The even charge Kondo effect can be realized experimentally in a setup similar to the one that was suggested to the odd two-channel Kondo effect[5]. In the ordinary setup (that was realized recently experimentally[6]) two noninteracting leads (L and R) and a large dot C are attached to a single-level small dot d. In the setup considered here we need more then one level on the small dot and require also to fulfil certain relations between the coupling constants (2). In addition, even when these symmetries are fulfilled, we still need to tune the magnetic field to be at a degeneracy point between the singlet and the triplet, and fine tune the dot-channel coupling

so that $J_\psi = J_c$. In practice this would be rather complected as many parameters should be tuned.

In a setup with two-valley double dot shown in Fig. 3 only one tunnel constant enters all exchange integrals. In this case the two-level dot d is formed by two single particle dots. Two dot levels $\epsilon_{1,2}$ are given by Eq. (18) and the single particle states of the small dot d are the symmetric and antisymmetric wave functions of single dots. As a result the coupling constants to the two levels are equal and the ratio (2) is fulfilled authomatically. This condition facilitates the fine tuning procedure.

B. Expected results

In order to identify the even channel Kondo effect one has first to identify the Kondo case when the finite reservoir C is open. This can be found by tuning the magnetic field, and the levels of the small dot so that $B_Z = 0$. Next when dot C is formed and tunneling of electrons between L and R to C is forbidden, one has to tune the coupling constants so that $J_\Psi = J_C$.

We will assume in the following the the coupling to lead L is much bigger than the coupling to lead R. Then as in the regular two channel Kondo effect[5,6] we expect that when $J_\psi = J_C$ differential conductance between the left and right leads $g(V,T)$ (with V the voltage and T the temperature) will follow the 2CK scaling law[10,20]:

$$\frac{2}{g_0} \frac{g(0,T) - g(V,T)}{\sqrt{\pi T/T_{K2}}} = Y\left(\frac{|eV|}{\pi T}\right) \qquad (21)$$

with $g_0 = e^2/h$ and the scaling function $Y(x)$ is given in Appendix B.

While $J_\psi \neq J_C$, the scaling law is:

$$\frac{1}{g_0} \frac{g(0,T) - g(V,T)}{(\pi T/T_\Delta)^2} = \text{sign}\,(\Delta) \frac{3}{2} \left(\frac{eV}{\pi T}\right)^2 \qquad (22)$$

with $\Delta = J_C - J_\Psi \ll J_\Psi, J_C$ and $T_\Delta = \Delta^2/J_\Psi^4 T_K$.

1. Effect of Magnetic Field

Unlike the single channel case, the magnetic field is a relevant parameter in the two channel case. We expect that the effective magnetic field in the even two channel Kondo case will operate in a way very similar to the operation of the ordinary magnetic field in the two channel Kondo case[10]. Notice though that the effective magnetic field B_Z [see Eq. (17)] is controlled by the real external magnetic field, by the position of the two levels and by the coupling constants.

Namely for $\sqrt{T_K T} \ll B_Z \ll B_\Delta = \left(\Delta/J_\Psi^2\right) T_K$ we have, for the linear conductance $g(V=0,T,B)$ the relation:

$$\frac{1}{2g_0} \frac{g(0,T,0) - g(0,T,B_Z)}{(B_Z/B_\Delta)^2} = -\text{sign}\,(\Delta). \qquad (23)$$

For $B_\Delta \ll B_Z \ll T_K$ the conductance is given by the Bethe-ansatz solution[10,21]

$$\frac{g(0,T,B_Z)}{2g_0} - 1/2 = a\,\text{sign}\,(\Delta) \frac{B_\Delta}{B_Z} - b\frac{B_Z}{T_K} \log \frac{T_K}{B}. \qquad (24)$$

with a and b of $\mathcal{O}(1)$.

V. SUMMARY

We studied the possibility to observe the two channel Kondo effect in multiple dots hetero-structures with even occupation in the presence of magnetic field. Like in case of single channel Kondo tunneling[2], we have found that magnetic field, which is as a rule detrimental for odd occupation Kondo regime, may induce the two-channel Kondo effect provided the Zeeman splitting compensates the exchange gap between singlet ground state and the state with spin projection oriented parallel to magnetic field. The effective spin of the small quantum dot in this case is 1/2 despite of the even occupation, so that the two channels provided by the large dot and the leads are sufficient to overscreen the dot spin. The exchange anisotropy which arises in presence of magnetic field disappears because of additional degeneracy in case when the small dot is split into two identical wells (Fig. 3). The two channel Kondo quantum critical physics may be stabilized by fine tuning of the parameters (i.e. coupling constants and external magnetic field).

VI. ACKNOWLEDGEMENT

APPENDIX A: SPIN ALGEBRA FOR DOUBLY OCCUPIED QUANTUM DOT

In case of strong Coulomb blockade, the low-energy part of the spectrum of complex quantum dots with even occupation consists of a system of singlet/triplet pairs (in the absence of the orbital symmetry, which may impose the Hund scheme of level occupation). These levels may be tuned by external magnetic field and gate voltages, and the accidental degeneracy arises as a result of level crossing. Under these conditions the notion of dynamical symmetry is extremely useful.[14] This symmetry characterizes not only the ground state of a system, but also the transitions between the eigenstates of quantum dot within a given energy interval (in our case this interval is determined by the Kondo temperature T_K). The simplest object, which possesses a dynamical symmetry is the S/T multiplet.

It is convenient to describe the eigenstates of a given Hamiltonian and the intra-multiplet transitions by the Hubbard operators $X^{\Lambda_1\Lambda_2} = |\Lambda_1\rangle\langle\Lambda_2|$. These operators obey the obvious multiplication rule $X^{\Lambda_1\Lambda_2}X^{\Lambda_3\Lambda_4} = X^{\Lambda_1\Lambda_4}\delta_{\Lambda_2\Lambda_3}$, from which the commutation relations may be easily derived.[7] The dynamical symmetry of S/T multiplet is characterized by two vectors $\mathbf{S}$ and $\mathbf{R}$ defined as

$$S^+ = \sqrt{2}\left(X^{u0} + X^{0d}\right), \quad S_z = X^{uu} - X^{dd}. \quad \text{(A1)}$$
$$R^+ = \sqrt{2}\left(X^{uS} - X^{Sd}\right), \quad R_z = -\left(X^{0S} + X^{S0}\right).$$

These two vectors obeying o_4 spin algebra generate $SO(4)$ symmetry group of spin rotator.[14] The dynamical symmetry of spin rotator is realized in Kondo tunneling near the point of accidental S/T degeneracy.

As is known,[2,14] the Zeeman splitting $E_Z = g\mu_B h$ reduces $SO(4)$ symmetry of CQD, when the external magnetic field induces the resonance $E_Z \approx \Delta_{ex}$. In this case one deals with a system of two quasi degenerate levels E_S, E_{Tu}, whereas two other components of triplet are quenched at low energy $\epsilon \ll E_Z$. The dynamical symmetry of this two-level system is described by a set of operators $\{X^{SS}, X^{uu}, X^{Su}, X^{uS}\}$, which form a closed $SU(2)$ algebra. To show this, one may introduce a pair of vectors $\mathbf{P}$, $\mathbf{Q}$ instead of (A1). The components of these vectors are defined as

$$P^+ = X^{uS}, \quad P^z = \frac{1}{2}(X^{uu} - X^{SS}),$$
$$Q^+ = X^{0d}, \quad Q^z = \frac{1}{2}(X^{00} - X^{dd}). \quad \text{(A2)}$$

These two vectors realize the mathematical possibility of factorization of the spin rotator symmetry group into a direct product, $SO(4) = SU(2) \times SU(2)$, so that each vector transforms as an effective spin 1/2. Only first of these vectors is relevant in the resonance condition $|E_Z - \Delta_{ex}| \sim T_K$, where T_K is the Kondo temperature, which characterizes cotunneling involving effective "spin" $\mathbf{P}$ with usual commutation relations $[P^i, P^j] = i\varepsilon_{ijk}P^k$, where i, j, k are cartesian components of this vector.

APPENDIX B: THE SCALING FUNCTION $Y(x)$

The function $Y(x)$ is given by:

$$Y(x) = -1 - \int_0^1 du \frac{3}{\pi} \frac{1}{\sqrt{u(1-u)^3}}$$
$$\times \left[\frac{|\log u|\sqrt{u}}{1-u}F(u)\cos(x\log u) - 1\right], \quad \text{(B1)}$$

$$F(u) = \frac{1-\sqrt{u}}{2}E\left(\frac{-4\sqrt{u}}{(\sqrt{u}-1)^2}\right)$$

with $E(x)$ the complete elliptic function.

It has the asymptotic form:

$$Y(x) \cong \begin{cases} c\,x^2 & \text{for } x \ll 1 \\ \frac{3}{\sqrt{\pi}}\sqrt{x} - 1 & \text{for } x \gg 1 \end{cases} \quad \text{(B2)}$$

with $c = 0.748336$.

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